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Effects of Geometer's Sketchpad on Spatial Ability and Achievement in Transformation Geometry among Secondary Two Students in Singapore

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Abstract: *Does the use of a common construction programme – the Geometer's Sketchpad - in different pedagogical settings have an impact spatial ability and achievement scores of students within concepts in transformation geometry? The subjects were 13 to 14 year-old students from a school with above-average ranking among Singapore schools. The software was employed differently in the three classes: In Class A, the approach adopted by the teacher was that of guided-inquiry where students explored concepts and made conjectures with extensive hands-on experience with the software; in Class C, the teacher's predominant role was that of an expositor and the students' role that of knowledge-recipients, and the software was used as a teacher's tool to demonstrate dynamically the properties of transformations; in Class B, the 'in between' class, the pedagogy of guided-inquiry in whole-class discourses was adopted but with the teacher manipulating objects on the projected screen as directed by the students. The results showed that spatial ability improved for all three classes with no significant difference between the classes although Classes A and B performed significantly better than Class C in the transformation geometry achievement test.*

Spatial Ability, Transformation Geometry and the *Geometer's Sketchpad*™ (*GSP*)

Although there is no one definition of the concept of "spatial ability" and various terms such as "visual", "imagery", "figure", and "mental diagram" have been used in describing the concept, the underlying common view is that spatial ability is a mental ability to form and manipulate objects visualised. One reason for the interest in spatial ability is that studies have shown positive correlations between spatial ability and mathematics achievement. Guay and MacDaniel (1977) found positive correlation in elementary grades while Fennema and Sherman (1977, 1978) found positive correlations in middle and high school grades.

Numerous studies have shown that spatial ability can be improved by training (Bishop 1980, Ben-Chaim, Lappan, & Houang, 1988). It would thus be of interest to educators to study how instructional programmes can be designed to help students improve their spatial ability. Since there is much in common between spatial thinking and transformation geometry (as they both involve construction and manipulation of mental images), "one might hypothesize that work with the latter would improve skills in the former" (Clements & Battista, 1992, p. 445). In particular, if teachers can demonstrate dynamically the motion involved in each transformation (be it a rotation, translation, reflection, enlargement, stretch or shear), they potentially could equip students with new visual-processing tools to perform transformations of mental images.

In a visual topic like transformation geometry, it is expected that the use of computer software which aids the visualization process would have a positive effect on student learning and attitude and this was borne out by results of a study by Woo-Tan as cited in Chong and Lim (1992) where specially designed software was used for teaching the topic. Since then, under the implementation of the IT "Masterplan" in Singapore schools, the *GSP* software has been introduced and is currently widely used. The *GSP* is especially suited to the teaching of transformation geometry because it (a) offers features that enable objects to be transformed on-screen, (b) allows the user to draw geometric primitives at ease and to measure accurately distances, angles and areas, (c) provides, with the click-and-drag feature, opportunities for students to elaborate on the attributes of figures constructed and (d) contains macros that allow animations of the motions involved in a transformation to be shown.

However, the act of bringing the *GSP* or any other computer software into the classroom alone will not guarantee a superior instructional environment. There is a need to learn *how* computing technology can be used to enhance learning in the classroom (Heid & Baylor, 1993).

Connell (1998) argued that technology usage must be tightly linked with the instructional programme. Using the constructivist philosophy as a guiding approach in teaching, Connell investigated the impact of technology usage in two separate classes where the technology was used in different ways. In one class, the teacher used the computers strictly as a presentation tool with the style of delivery more in line with a behaviourist model. In the other class the constructivist approach was adopted as a guiding philosophy in the classroom. As such, the first class was known as a technologically-misaligned classroom (TMC). In the other class, the computers were used as a tool for students' exploration, and was thus known as a technologically-aligned classroom (TAC).

This paper reports on a study investigating how different instructional approaches with common construction software (*GSP*) affected students' spatial abilities and their concept formation of ideas within the domain of transformation geometry.

Methodology

The sample consisted of 122 Secondary Two (13 to 14 -year-old) students from an above average school in that it ranked consistently among the top 40 out of the more than 230 secondary schools in the Singapore Ministry of Education's ranking exercise. The students were from the Express stream to which is assigned the top half of each cohort when students leave the primary schools to enter secondary schools. Due to practical time-table constraints, it was not possible to re-assign the students to mathematically equivalent classes and thus a limitation of the study is that three intact Secondary Two classes (Class A, Class B and Class C) were used. The number of boys and girls in Class A, Class B and Class C were 28, 25, 26 boys and 13, 17, 13 girls respectively. Based on the Secondary Two first semester's examination mathematics scores, Class C showed significantly lower scores – mean scores for Class A, Class B and Class C were 71.5, 73.7 and 66.3 respectively. This examination tested on topics in Algebra and Arithmetic.

The learning environment in Class A was modeled to be in line with the "pedagogy of guided inquiry". It is

> an approach to curriculum and instruction which gives the teacher the responsibility for introducing content in a way that is illuminated and modified in response to students' way of thinking about that content. The teacher defines the focus of inquiry by posing problems for the class while students take an active part in acquiring knowledge by generating not only answers, but ways of thinking about problems, definitions of the terms of the discourse, and analyses of alternative solutions (Lampert, 1988, p.1).

Teachers who adopt such a pedagogy set curriculum goals (like any other regular classrooms), but instead of advocating teacher's direct feeding of information, promote a social discourse among students that encourages reasoning that zigzags between observations and generalisations. The *GSP* was thus used as a tool for students' exploration and conjecturing in this class.

Class C modeled the learning environment of a regular local classroom, where the teacher assumed the role of an expositor and authority. In this class, the *GSP* was a teacher-directed demonstration tool used in a direct-instruction setting. Students recorded results presented on screen and reproduced procedural steps with paper-and-pencil.

Class B modeled the "in between" of Class A and Class C. The teacher promoted a spirit of inquiry in the students but the *GSP* was used as a teacher-controlled tool to elaborate on the students' observations and to verify their conjectures while the students viewed the displays on the screen at the front of the class. Figure 1 below illustrates the model description of the three classes.

The assignment of the classes to the treatment types was done in a random fashion.

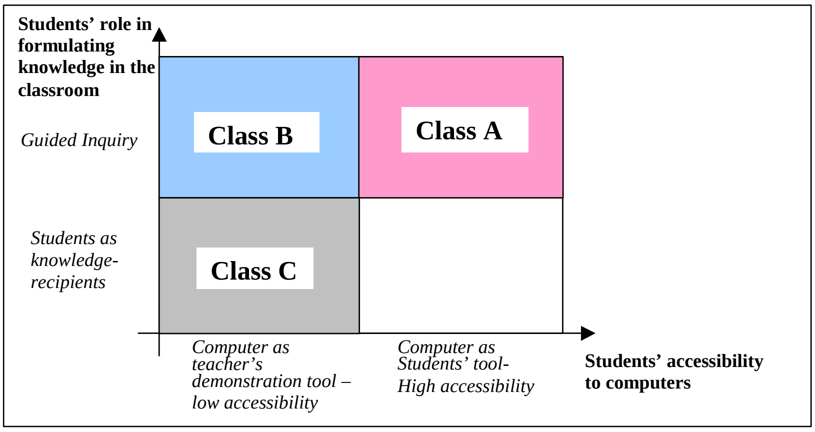

Figure 1. Model of learning environments in the three classes

As the underlying pedagogy matches the way the computer was used, both Class A and Class C, although in different ways, are TACs using Connell's terminology; in Class A, it was an alignment to constructivism and in Class C it was an alignment to behaviorism. In Class B, since the computer was teacher controlled but the method of teaching founded on constructivism underpinnings, it was a TMC.

The fourth 'window' in Figure 1 above would describe an instructional environment where the teacher employs a direct-instruction method, with students using the computers to practise prescribed construction procedures demonstrated by the teacher. As there were only three Express stream classes at the Secondary two level in the school, this classroom environment was not modeled.

The treatment was carried out over three weeks and began on the first day of the fourth school term just after a one week recess. There were seven periods of mathematics in each week, each lasting between 30 to 35 minutes. Although lessons which allow students to work on computers often require more time than regular direct-teaching lessons, in this study all three classes undertook the module over the same period of time. This choice of duration was dependent on the number of periods pre-allotted to the topic at hand in the scheme of work specified by the school. During these 21 periods, the classes were exposed to "motion geometry", which included concepts in reflections, rotations, translations and enlargements.

As the students in these classes had no prior formal experience with the *GSP*, there was a need "for students to work through the mechanics of learning a new software" to "build the confidence needed to persist in more complex geometry problems" (Pokay & Tayeh, 1997, p.120). Prior to the treatment, each class was given a 3-hour *GSP* preparatory course to provide them with hands-on familiarisation with the tools within the software. The contents of the course and the manner in which it was conducted were identical for all the three classes and they were conducted by one of the authors. The preparatory course was conducted within the school recess just before the treatment period.

Class A Treatment

This class was taught by one of the authors during the treatment period. Of the twenty-one 35-minute periods of the treatment duration, 12 were double-period sessions and 9 were single-period sessions. As the time taken to move the students between the computer lab and classroom was considerable, students were brought to the computer lab only during double-period sessions. It was thus natural that the students' exploration, induction, conjecturing and verification was conducted in the computer labs during the double-period sessions (where the *GSP* was available to the students) and the discussion, whole-class explanation and paper-and-pencil construction was done in the classroom during the single-period sessions. At the lab sessions, students worked on the computer in pairs.

The role of the teacher during class times was that of a social discourse authority. A social environment conducive of students' inquiry, where students were free to make observations, ask questions, make conjectures, challenge conjectures, and where students were led to focus on the discussions at hand was maintained. A general structure of activities with targeted goals as a form of guide for students to engage in exploration was provided. The tasks in the activities did not require rigid adherence to step-by-step procedures, but encouraged students to follow varied experimentation paths. So, as the students manipulated on the primitives to observe properties and performed on-screen constructions, the direction of learning, though zigzagging differently for different students, was heading towards the desired curricular goals.

Sample lesson – on "Rotation" in Class A

The content introduction took the form of a few simple "administrative" instructions from the teacher to the class on which files to open, what objects to drag and their responsibility to make observations. The students then proceeded on their own in their pairs, using the mouse, to move objects on their monitors and observe the resulting movements on the screen. A typical student-pair's display is shown in Figure 2 below (the trace lines were not on the students' screens but created here to indicate motions).

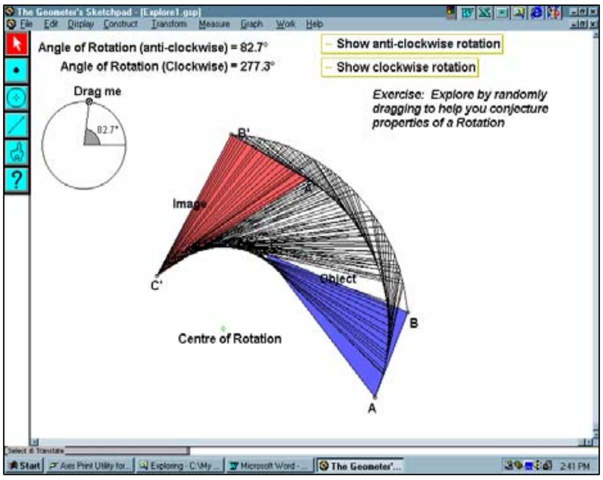

Figure 2. Students monitor display in Class A (adapted)

The students could click-and-drag on any whole or parts of all the geometrical objects on screen. One could move the "drag me" point to vary the angle of rotation between the object and image. Also the object/image could be moved (by dragging points, line segments or the interior) to see how the corresponding image/object would vary. The centre of rotation could be moved to see how the image moved accordingly. While all these movements took place, the measures on screen were updated "live" in the sense that they would change according to the relational changes of the screen objects. In addition, there were the "Show clockwise movement" and "Show anti-clockwise movement" buttons which could be depressed to view the dynamic clockwise and anti-clockwise motions respectively of the object to the image. The design of these student exploration templates was created with the least possible constraints in motions so that it might serve as adequate tools for the students to obtain sufficient generalisability to observe relations and make inductions on those observations.

Students were then invited to share their observations with the class. All these observations were written on the first section of the board without judgment. Students were then asked to look at the observations and make refinements or conjectures on those observations. These were recorded on the second section of the board with their names beside their conjectures. It was at this stage of the lesson where disagreements over the correctness of the conjectures amongst the students were anticipated and where judgments of truth and falsity of these conjectures were expected to come from the teacher – the traditional intellectual authority. It was however, to the teacher, the most opportune time to throw the authority back to the students (thus legitimising their inquiry as a means of learning mathematics) by telling the class to "Prove it!" The teacher, however, could not, in the limited time provided, instruct the investigations of all the conjectures on board. It was here that the teacher selected, on behalf of the students, conjectures the results of which would be in line with curricular agendas, thus connecting the students' inquiry with curricular goals. These conjectures were then written on the third section of the board.

The students then returned to the same templates on their screen and employed the tools on the *GSP* to verify the conjectures directed by the teacher. Constructions made are to be 'drag-resistant' (Hoyles & Noss, 1994). Students' work files were then saved in diskettes. This point marked the end of a double-period lab session. Their diskettes were then collected for evaluation. Interesting points that arose from students' work were raised for the subsequent discussions in the classroom.

Class B Treatment

The class was taught by the same teacher as Class A during the treatment period. All the lessons were conducted in the classroom. The approach in pedagogy and the materials used for this class were similar to those in class A. The number of worksheets and textbook exercises given were also identical. The difference lay in how the students interacted with the software. Where students could directly access the *GSP* for exploration in Class A, students in Class B conveyed their intention of what visual motions they wanted to explore through the "agent " of the teacher, who performed the manipulations for the whole class to view on the projected screen in front of the class.

With this teacher-as-intermediary approach to software use, the interaction time with the computer in this class was significantly less compared to that in class A. The additional time was used to do more monitoring of seatwork as students attempted the worksheets in class. Compared to class A, students in class B had more paper-and-pencil construction practices under the supervision of the teacher.

Sample lesson – on "Rotation" in Class B

The teacher introduced "rotation" to the class by projecting Figure 2 (p. 5) on the screen in front of the class. As the students' attention was directed to the screen, the teacher moved the mouse on the points and figures one after another on his computer monitor to show how the movement of certain points or figures would be dynamically linked to the movement of other parts. The "Show clockwise rotation" and "Show anticlockwise rotation" buttons were also repeatedly depressed so that the students could visualise the rotation movements of the object to the image. Students were free to request for certain dragging motions to be repeated. As much as the interest of the class was focused on the screen, the teacher repeated the movements to help the students form generalisations of properties of "rotations."

Students were then invited to make observations of what they saw on the screen. The procedure of recording students' observations on the first section of the board, followed by refinements of these observations to conjectures and subsequently a call from the teacher to "Prove it!" was similar to that conducted in Class A. Students were found to disagree on one another's conjectures, but the teacher, refraining from making any evaluative judgments, invited students to suggest ways where their conjectures or refutations could be verified by the tools in the *GSP*. Students' 'directions' of what was to be done on the screen, such as measuring line segments, were demonstrated by the teacher and evaluated by the class. In the process, the teacher modified students' informal language – usually complemented by pointing at objects on screen – to terms common in mathematical use. Particular attention was given to students checking that their assertions were true by applying the "drag-test." Conjectures that were 'proven', as affirmed by the class, were written on the third section of the board as properties that the class could apply subsequently to problems.

The follow-up to the whole-class exploration segment was centered on students' attempts on teacher-prepared worksheets (identical to those used by Class A). Unlike in Class A, the students in Class B had sufficient time to complete the paper-and-pencil worksheets and to discuss problems encountered in their homework assignments. This 'extra' time that Class B had over Class A was due to the relatively less time spent on the computer as compared to Class A, where students worked directly on the machines.

Class C Treatment

The class was taught by two resident teachers during the treatment period. That there were two teachers in Class C was the arrangement of the school's administration. Such an arrangement was in place before this study commenced. The two teachers in the class took turns to instruct the class. While one teacher was leading in the whole class teaching, the other would assist by helping in the computer hardware and in maintaining a level of on-task attention among the students. The teachers adopted a direct-teaching method common in Singapore schools. The teachers delivered a pre-planned agenda to the class. They would demonstrate the properties of the various transformations on the projected screen in front of the class and the students would observe and ask questions for clarifications along the way. The worksheets used by the students were mainly fill-in-blank type, where observations and results obtained from structured teacher demonstrations on the *GSP* were recorded there. Intermittently, students would be invited to participate with providing answers for blanks to be filled in the worksheets. Constructions were also presented on *GSP* followed by step-by-step constructions on board using the marker. Students followed the constructions using paper-and-pencil and subsequently reproduced the steps on other practice questions.

The textbook from where the exercises given to students were taken was the same as that used in Class A and Class B. Other worksheets similar to those used by the other classes in terms of the skills required to do the constructions were also given as exercises to students.

Sample lesson – on "Rotation" in Class C

The teachers introduced 'rotations' by showing the motions of rotation on a screen in front of the class. That was followed by an exposition of the labels used for different objects on the screen, which included the centre of rotation, the object and image of rotation and the angles of rotation in both the clockwise and the anti-clockwise directions. By dragging these objects one at a time, the teacher pointed out to the students how other related objects are dynamically updated. Further explication of the properties of 'rotation' was uncovered via a question-and-answer format using a worksheet adapted from a guidebook (Soon & Yap, 1999). The teacher invited answers from the students. Answers were evaluated by the teacher and the class moved on from one question to the next. The teachers concluded the introductory lesson by summarising salient points relating to the attributes of a rotation.

Instruments and interviews

The Wheatley Spatial Ability Test™ (WSAT)

Pre- and Posttests using the WSAT on the three classes were administered. The WSAT tests students' ability to perform mental rotations as a dimension of spatial ability. The test consists of 100 items divided into 20 sets of five items. In each set, the student is required to compare a given planar geometric figure with another five congruent figures, and the student's response for each item in the set is either "yes" or "no" - "yes" if they differ by a rotation and "no" if a reflection is required. The reader is referred to Clements, Battista, Sarama, and Swaminathan (1997) for sample items from the WSAT. The pretest was administered before the commencement of the *GSP* preparatory course and the posttest was administered at the end of the last lesson in the treatment period. Sixteen students (7 from Class A, 7 from Class B, and 2 from Class C) did not attempt the pretest as they were absent from the preparatory class.

The WSAT has been used widely in many research studies (more recent studies include Brown, 1993; Brown & Wheatley, 1989, 1990, 1991; Wheatley, Brown & Solano, 1994) and has been found to be a valid and reliable test with a high internal consistency. To check that cultural differences do not significantly change the internal consistency of the test in this study, the Cronbach's alpha (the generalised form of the K-R coefficient) was computed for the Pre-WSAT, giving a value of 0.975 (N = 106). As a norm, the duration of this test for students in elementary grades is 8 minutes. As the students in this study were at the Secondary Two level (the age-equivalent of Grade 8), the recommended duration of the test was reduced to 7 minutes (Wheatley, personal communication, June 2000).

The Motion Geometry Test (MGT)

The MGT was administered at the end of the treatment duration to all the three classes. The test was designed to assess the students' level of achievement of curricular targets as stipulated by the syllabus at the end of the 21 periods of lessons under different instructional environments. Also in line with the mode of assessment typically found in the Singapore Education System, a paper-and pencil test format was used. A pre-treatment MGT was not administered as the students did not have experience with transformation geometry prior to treatment.

The duration of the test was one hour. The test consisted of 20 items, each item testing a specific skill competence corresponding to a specific instructional objective spelt out in the national curricular objectives for students' achievement in transformation geometry at secondary two level. The Cronbach's alpha for estimating reliability of the MGT for this study sample was 0.75 (N = 121; one student from Class C was absent on the day the MGT was conducted).

One-to-One Assessment

An audio-recorded, one-to-one interviewer-student, on-screen response assessment was conducted 5 weeks after the end of the treatment. The purpose of these sessions was to observe (in a depth not easily revealed by conventional paper-pencil tests) whether there were differences in the students' approaches in tackling novel problems in transformation geometry and in their level of geometric and spatial reasoning which had arisen out of their exposures to different didactic environments using the *Geometer's Sketchpad*.

The on-screen assessment did not require the students to manipulate the objects on-screen. They were also not permitted to do so. This requirement in the design of the assessment was to prevent the possible advantage that students in Class A had over students in the other classes as they spent more time working with the software hands-on.

Five students were selected from each of the three classes. For the purpose of ensuring that the sampling spanned a range of abilities, three bands, in terms of spatial ability, of the students were determined based on their WSAT Pre-test scores. Scores less than 55 were classified under band I; scores between 55.5 and 70 were classified under band II; and scores beyond 70.5 were placed under band III. Out of the 5 students from each class, one was from band I, one from band II and three were from band III. This 1,1,3 selection from the respective bands was chosen to correspond to the approximate ratio of 1:1:3 of the number of students whose Pre-test scores were in the respective bands. The choice of student(s) within the bands was random and was undertaken by the resident mathematics teachers of the respective classes.

Each session with the individual students lasted between 15 and 25 minutes. Students were presented with four questions. The objects and images in the four questions were built on the same diagrammatic structure so as to reduce students' difficulties contributed by variability in geometric contexts. All four questions were

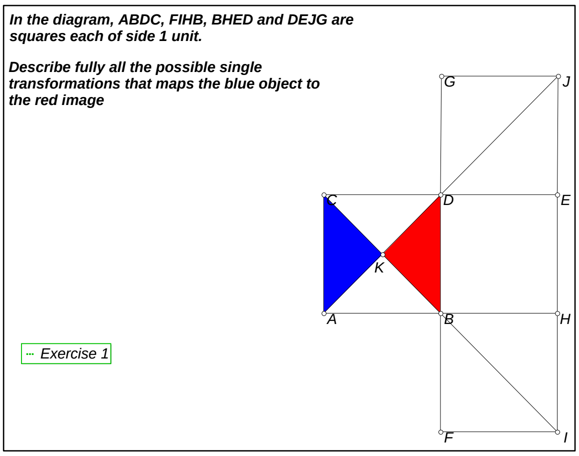

Figure 3. A sample question in the interviewer-student session

presented to the students on the monitor screen of a portable computer. Each question presented a diagram showing the object and image of a single transformation (one of the four – rotation, reflection, enlargement or translation). The students were to describe completely as many single transformations as possible to map the object to the image. A sample question taken from the on-screen items is shown in Figure 3. In each of the exercises, students were told that they were to describe all the possible single transformations mapping the object to the image. While the concept of transformation geometry requires the point-to-point correspondences between object and image, the focus of this exercise was principally to check the students' familiarity with description of different transformations.

Results and Discussion

WSAT Scores

Paired-sample t-tests at the 5% level were conducted on the pre- and post- WSAT scores of each class to determine possible treatment effects on WSAT scores. The sample size for each class was adjusted to take into consideration the students who did not take the pretest. A summary of the mean scores and the respective t-statistic are provided in Table 1 below.

Table1.
WSAT pre- and posttest scores

	Class A		**Class B**		**Class C**	
	Pre	Post	Pre	Post	Pre	Post
Mean	80.1	91.3	81.0	90.2	88.5	93.9
SD	20.1	12.9	20.7	16.3	11.5	12.0
Mean difference	13.5		9.5		2.7	
t-statistic	4.96		4.20		2.37	
Significant inc.	Yes		Yes		Yes	
Effect size	0.85		0.75		0.39	

Since retesting with the WSAT is not likely to yield substantive increases (Clements, Battista, Sarama, & Swaminathan, 1997), the significant increases found in this study may therefore be credited to treatment effects. The result shown here also contributes to strengthening the conjecture that work on transformation geometry improves spatial ability.

ANCOVA was applied to test differences in posttest scores between the classes after adjusting for differences in the pre-test scores. The F-value result (at 0.85) showed no significant difference between treatments at the 5% level. This result seems to indicate that the differences in the learning approaches adopted by different classes in the use of the software did not feature as a significant factor in the increases in the WSAT scores.

The Motion Geometry Test (MGT)

The maximum score for the MGT is 40 (2 times 20 items). The mean scores in the MGT for the classes A, B and C are 35.9, 36.4 and 31.9 respectively. To determine significant difference in the MGT scores between the classes, a one-way ANOVA was conducted with a resulting *F*-value of 11.140, which shows highly significant difference, even at 1% level. Posthoc testing using the Tukey HSD indicated that the MGT scores in both Class A and Class B were significantly higher than scores in Class C, but did not indicate any significant difference between Class A and Class B scores.

A detailed item-by-item analysis was also carried out. Only in items 12, 13, 14 and 20 were there significant differences in performance. In each of these four items, there was no significant difference between Class A and Class B scores but significant A-C and B-C differences. Item 12 of the MGT, shown in figure 4, is a typical example for a brief discussion on the differences in students' performance between Class C and the other two classes.

Of the 38 students who attempted the test in Class C, 18 of them wrongly identified the transformation in item 12 as "reflection about the *y*-axis". This contrasted sharply against six in Class A and 4 in Class B who made the same error. There is thus an indication that a larger number of the students in Class C had not developed correct concept-images of "rotation", possibly confusing them with those of "reflection". Allowing the students to explore figures, via conjecturing and testing of their conjectures, in more generalised visual settings using the *GSP* (as was done in Classes A and B) appeared to have enabled the students to form better concept-images. The results here strengthen the cause of employing a pedagogy which encourages students' inquiry (as was in classes A and B) as a *feasible* (and possibly more effective) approach to helping students attain the goals stipulated in a traditional curriculum.

One-to-One Assessment: Analysis of students' responses to exercise 1

For ease in referencing, subjects were labeled Xi, where X indicates the treatment class and i the student, with i = 1 corresponding to the student selected from band I, i = 2 from band II and i = 3, 4, 5 from band III. This exercise (see Figure 3) required the subjects to identify all the possible single transformations mapping the object to the image shown. They were also to describe the transformations fully. The two expected correct responses were a 180 degrees rotation about point K and a reflection about the line passing through K and parallel to AC.

Δ**ABC** is mapped onto Δ**A'B'C'** by a single transformation **P**. Describe fully the transformation **P**.

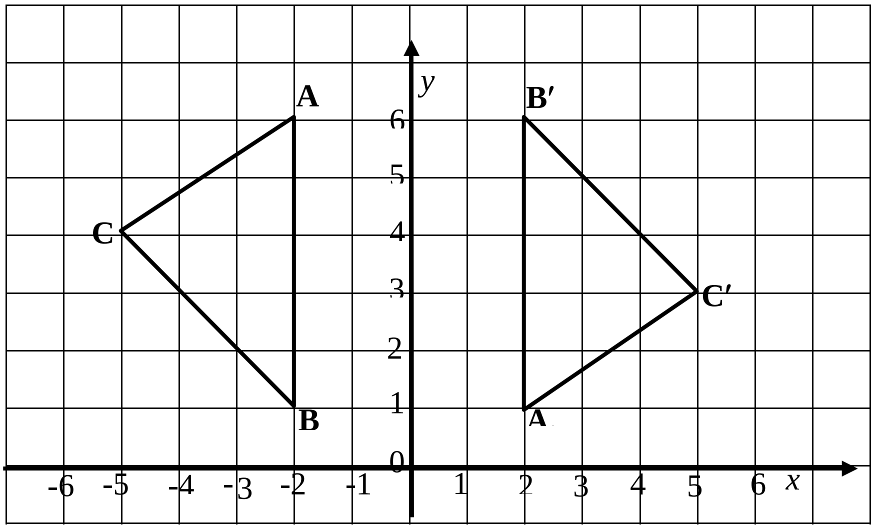

Figure 4. Item 12 MGT

Three students from Class A (A2, A3 and A4) were able to give the correct and full descriptions of the two required transformations with little prompts from the teacher. Student A1 could only identify (but did not describe) the reflection. Student A5 was able to identify both the rotation and reflection but could not describe the line of reflection.

In sharp contrast to the responses by students from Class A, none from Class B were able to fully describe both the transformations, despite registering longer strings of interviewer-student dialogue wherein prompts by the interviewer were given. Student B1 identified both the transformations but did not provide descriptive details. Student B3 did not state the centre of rotation despite prompts for more details in his description. None of the five students in Class B were able to describe the line of reflection with reference to the objects on the screen. Students C2, C3 and C4 could, with some prompting, correctly describe the two transformations. Students C1 and C5 could not, like a majority of others in the sample of 15 students, describe the line of reflection without pointing to the screen for visual support to their verbal responses.

The data above showed that the students had little difficulty with identifying the reflection and the rotation, but when precision in detailing each transformation was required, they had varying difficulties, with students from Class A showing greatest ability to describe to the required precision and with least guidance.

Further analysis of the students' responses in describing the 'line of reflection' also revealed qualitative differences in Class A students' responses. When attempting to describe the line of reflection, students in Class B could not go beyond relying on visual reference (such as by pointing on the screen) to identify the line and lacked semantic tools to describe the object of reference, as indicated by responses such as "reflection at the line K (B2)", "line K (B3)", "line pass through K (B4)", "reflection line here . . . bisecting K (B5)". Although a few in Class C could describe the transformations with satisfactory details, the language used reflected a lack of ability to abstract properties in-built within the object provided, as can be observed from the responses "A line crossing K . . . *vertically* crossing K (C2)", and "mirror line at K . . . *vertical* line cuts K (C3)". The use of the term 'vertical' is tied to the orientation of the drawing (which is a variant property) and would not hold if the drawing were not stereotypically 'upright'. In this sense, their descriptions were less flexible. In contrast, student responses from Class A were "the line of reflection is at K . . . the perpendicular bisector of AB (A2)", "line pass through the centre of A and B and the centre of C and D (A3)", "reflection about the line passing through the centre and parallel to BD (A4)". The mathematical language used was precise and reflective of an understanding of invariant properties intrinsic in the figures provided. That the responses given from these students from Class A were so varied (and yet correct) underlay a richness in the development of the mathematical language and geometric understanding within these students in the process of their learning.

The analyses of the students' responses in other exercise tasks similarly point to consistently better answers coming from Class A students. They indicated that these students have better developed concept-images characterised by an understanding of critical properties of each transformation. These mental representations were also more flexible and could be manipulated and evaluated. Fewer students from the other classes demonstrated the same abilities.

Conclusion

The research question on how different modes of use of the *GSP* impacts students' learning is answered differently by the different instruments used in this study. The WSAT scores did not point to any significant differences in pre-posttest increase between the three classes. The MGT scores, however, separated Classes A and Class B from Class C. As well, further analysis for a more in-depth picture with the interview assessment, Class A seemed to stand apart from Classes B and C in terms of strength of concepts formed in the students.

Regardless of underlying pedagogy, classes whose teachers included a substantive use of the *GSP* in an instructional module on transformation geometry made significant improvement in their WSAT, indicative of improvement in spatial abilities. As *GSP* is a relatively new tool in for use in Singapore classrooms, the potential of the software in enhancing geometric thinking and spatial reasoning should be an area of further research.

As well, apart from the method of technology use, the pedagogy of guided inquiry is feasible with regards to meeting curriculum targets, as attested by the significantly better performance of the MGT scores in classes which employed such an approach to classroom discourse. This observation challenges the still popular belief that the most effective way for students to perform in mathematics is by "drill and practice" alone. However, the method of technology use and the method of instruction do not appear unrelated. Connell's argument of a tight linkage between the constructivist approach to teaching and technology use was supported by the task responses of students from Class A. This study provides promise to teachers who desire to realise the theoretical ideals of a constructivist classroom environment without compromising on students' assessment results.

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